

Chapter 1

TRANSPORT IN LUTTINGER LIQUIDS

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Abstract

We compute the transport properties of one dimensional interacting electrons, also known as a Luttinger liquid. We show that a renormalization group study allows to obtain the temperature dependence of the conductivity in an intermediate temperature range. In this range the conductivity has a power-law like dependence in temperature. At low temperatures, the motion proceed by tunnelling between localized configurations. We compute this tunnelling rate using a bosonization representation and an instanton technique. We find a conductivity $\sigma(T) \propto e^{-\beta^{1/2}}$, where β is the temperature. We compare this results with the standard variable range hopping (VRH) formula.

Keywords: Luttinger liquid, creep, conductivity, variable range hopping, disorder

1. Introduction

Since the discovery of Anderson localization [1], impurity effects in electronic systems have always been a fascinating subject. Although our under-

standing of the properties of noninteracting disordered electronic systems is now rather complete [2–5], the interacting case is still largely not understood. Indeed the combined effects of disorder and interactions leads to a reinforcement of both the disorder and interactions effects and complicates greatly the physics of the problem [6–8].

One dimensional systems are an extreme realization of such a situation. On one hand, even for noninteracting systems disorder effects are extremely strong and all states are exponentially localized [9, 10]. On the other hand for the pure system, interactions have an extremely strong impact and lead to a non-fermi liquid state known as a Luttinger liquid [11]. One can thus expect a maximal interplay of disorder and interactions there. However, in one dimension, good methods such as bosonization exist to treat the interactions, so one can expect to have a more complete solution even in presence of disorder.

We examine here the transport properties of such Luttinger liquids.

2. Model

For simplicity we focuss here on spinless electrons. We consider an interacting electronic system. Using the standard boson representation [11] the Hamiltonian of such a system is

$$H = \frac{1}{2\pi} \int dx u K (\pi \Pi)^2 + \frac{u}{K} (\nabla \phi)^2 - \frac{1}{2\pi\alpha} \int dx \xi^*(x) e^{i2\phi(x)} \quad (1.1)$$

where $\xi(x)$ is a (complex) random potential representing the backward (i.e. close to $2k_F$) scattering on the impurities. $\xi(x)$ is taken to be gaussian and uncorrelated from site to site

$$\overline{\xi(x)\xi^*(x')} = D_b \delta(x - x') \quad (1.2)$$

and all other averages are zero. The field ϕ is related to the density of fermions by

$$\rho(x) = -\frac{1}{\pi} \nabla \phi(x) + \frac{1}{2\pi\alpha} e^{i(2k_F x - 2\phi)} + \text{h.c.} \quad (1.3)$$

and the current is simply $J = \partial_\tau \phi / \pi$.

In (1.1) the interaction effects among the electrons are hidden in the two Luttinger parameters u and K . u is the velocity of charge excitations. In the absence of interactions u is the Fermi velocity $u = v_F$. K is a dimensionless parameter, controlling the decay of the various correlations. $K = 1$ in the absence of interactions and $K < 1$ for repulsive interactions. α is a short distance cutoff of the order of the lattice spacing.

An external electric field E thus couples as

$$\int dx A J = -\frac{1}{\pi} \int dx E \phi(x) \quad (1.4)$$

In the absence of disorder, the electric field makes the phase ϕ grow with time. As can be seen from the Kubo formula, for the pure system the conductivity is infinite. This corresponds physically to the sliding of the electronic “charge density wave” (1.3). Disorder pins the electronic density. Such a pinning corresponds in the electronic language to the Anderson localization of the electrons [12, 13].

3. Transport at intermediate temperatures

The phase diagram of (1.1) has been extensively studied and we refer the reader to [14, 11, 15] for details. Renormalization group equations for the disorder D_b can be written from the action

$$S/\hbar = \int dx d\tau \left[\frac{1}{2\pi K} \left[\frac{1}{u} (\partial_\tau \phi)^2 + u (\partial_x \phi)^2 \right] - \frac{\xi^*(x)}{2\pi\alpha\hbar} e^{i2\phi(x)} + \text{h.c.} \right] \quad (1.5)$$

These equations are

$$\begin{aligned} \frac{dK}{dl} &= -\frac{K^2}{2} \tilde{D}_b \\ \frac{d\tilde{D}_b}{dl} &= (3 - 2K) \tilde{D}_b \\ \frac{du}{dl} &= -\frac{uK}{2} \tilde{D}_b \end{aligned} \quad (1.6)$$

where

$$\tilde{D}_b = \frac{2D_b\alpha}{\pi u^2} \quad (1.7)$$

For $K < 3/2$ the disorder is a relevant variable and leads to localization. This of course includes the noninteracting point $K = 1$.

Using the RG, one can extract various physical quantities for the disordered Luttinger liquid. For example, one can extract the localization length. Let us renormalize up to a point where $K^2 \tilde{D}_b(l^*) \sim 1$. The true localization length of the system is given by

$$\xi_{\text{loc}} = e^{-l^*} \xi_{\text{loc}}(l^*) \quad (1.8)$$

but if $\tilde{D}_b(l^*) \sim 1$ the localization length of such a problem is of the order of the lattice spacing. Thus,

$$\xi_{\text{loc}} \sim \alpha e^{-l^*} \quad (1.9)$$

One can then integrate the flow to get l^* . The result depends on the position in the phase diagram. When one is deep in the localized phase (far from the transition) one can consider K as constant and thus

$$\tilde{D}_b(l) = \tilde{D}_b(l=0) e^{(3-2K)l} \quad (1.10)$$

Thus,

$$\xi_{\text{loc}} \sim \alpha \left(\frac{1}{K^2 \tilde{D}_b} \right)^{\frac{1}{3-2K}} \quad (1.11)$$

One can also extract the frequency and temperature dependence of the conductivity [14]. Let us here look at the temperature dependence by a very simple technique. The idea is simply to renormalize until the cutoff is of the order of the thermal length $l_T \sim u/T$ corresponding to $e^{l^*} \sim l_T/\alpha$. At this lengthscale the disorder can be treated in the Born approximation. As the conductivity is a physical quantity it is not changed under renormalization and we have

$$\sigma(n(0), \tilde{D}_b(0), 0) = \sigma(n(l), D(l), l) = \sigma_0 \frac{n(l) \tilde{D}_b(0)}{n(0) \tilde{D}_b(l)} = \sigma_0 \frac{e^l \tilde{D}_b(0)}{\tilde{D}_b(l)} \quad (1.12)$$

where $\sigma(n(l), \tilde{D}_b(l), l) = \sigma(l)$ and $n(l)$ are, respectively, the conductivity and the electronic density at the scale l . $\sigma_0 = e^2 v_F^2 / 2\pi \hbar D_b$ is the conductivity in the Born approximation, expressed with the initial parameters. If one is deep in the localized phase, one can again retain only the RG equation for the disorder and consider K as constant and one has

$$\sigma(T) \sim \frac{1}{\tilde{D}_b} T^{2-2K} \quad (1.13)$$

This result is schematized in Fig. 1.1.

4. Creep

Although one can use the RG to get the behavior of the conductivity for $T > u/T_{\text{loc}}$, it cannot be used below this energy scale since the flow goes to strong coupling. In order to determine the transport properties at lower temperatures, we compute the tunnelling rate between two static configurations of the system. The details can be found in [16], so we will recall here only the main steps and results. We first use the RG equations to reach a lengthscale at which the disorder \tilde{D}_b becomes of order one. This lengthscale corresponds to having a “lattice spacing” that is now of the order of the localization length of the system. Writing the disorder $\xi(x)$ as

$$\xi(x) = |\xi(x)| e^{i2\zeta(x)} \quad (1.14)$$

we now see that the disorder is minimized if on each “site” the phase ϕ takes the value

$$\phi(x) = \zeta(x) + \pi n_x \quad (1.15)$$

where n_x is an integer. The integer n_x have to be chosen in order to minimize the elastic term $(\nabla \phi(x))^2$ in (1.5). Thus in the absence of the quantum term

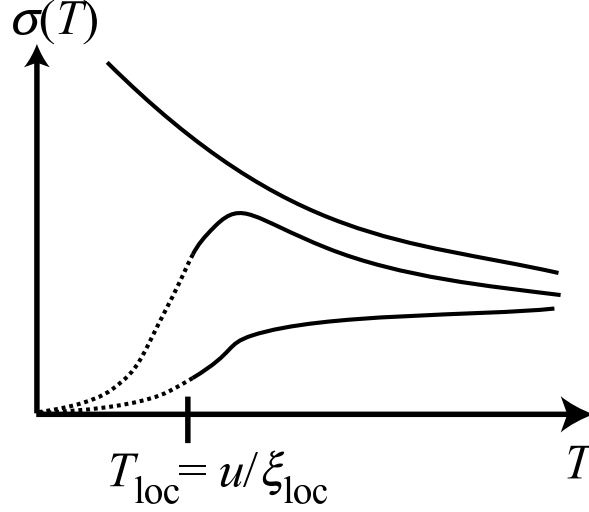


Figure 1.1. Temperature dependence of the conductivity. For $K > 3/2$ (top) the system is delocalized and the conductivity increases with decreasing temperature. For $1 < K < 3/2$ (middle) the system is localized but the conductivity starts increasing with decreasing temperature. The renormalization of K due to disorder pushes the system to the localized side forcing the conductivity to decrease with decreasing temperature. For $K < 1$ (bottom) the conductivity decreases with temperature even at high temperatures. Below temperatures of the order of u/ξ_{loc} , the system is strongly localized and the conductivity decreases exponentially (see text). This part (dashed line) cannot be extracted from the RG.

Π^2 in the Hamiltonian the system is completely characterized by the set of integer numbers n_x [17]. The electric field (1.4) wants to make the phase grow. Thus in the presence of the quantum term in the action the phase will tunnel between the optimal configurations described by (1.15). This corresponds to an increase of n_x by one in some region of space. In order to compute the action corresponding to such a tunnelling process one uses an instanton technique as introduced in [18, 19] for the pure system and [16] in the presence of disorder. The size L_x in space and L_τ in time of the instanton are determined by extremizing the action. One finds

$$L_x^{\text{opt}} = \sqrt{\alpha/\epsilon} \quad , \quad L_\tau^{\text{opt}} = 1/(2\epsilon) \quad (1.16)$$

where

$$\epsilon = \frac{2K^* \alpha E}{\pi u^* \hbar} e^{2l^*} \quad (1.17)$$

and $*$ denotes quantities as the scale ξ_{loc} .

At zero temperature one can thus obtain the action of the instanton as a function of the electric field. This leads to the tunnelling rate

$$P \sim \exp\left[-\frac{1}{\sqrt{2}}\left(\frac{\pi}{K^*}\right)^{3/2}\left(\frac{\Delta}{E\xi_{\text{loc}}}\right)\right] \quad (1.18)$$

where we have introduced a characteristic energy scale $\Delta = \hbar u^*/\xi_{\text{loc}}$ associated with the localization length. Note that u^*/ξ_{loc} is the pinning frequency [20]. The expression (1.18) leads to a non-linear response. The linear conductivity is zero, and such a process is the analogue of the creep for classical systems [21–23]. In this case the system is able to overcome barriers by quantum tunnelling, instead of thermal activation for the classical case.

For finite temperatures, the maximum size of the instanton in time is $L_\tau < \beta \hbar u^* e^{-t^*}$. If the electric field becomes too small the action associated with the tunnelling process thus saturates. One thus recovers a linear response that is given by

$$\sigma(T) \propto e^{-\frac{S^*}{\hbar}} = \exp\left[-\frac{\pi}{K^*}\sqrt{2\beta\Delta}\right] \quad (1.19)$$

5. Variable range hopping

The expression (1.19) leads to the same temperature dependence than the famous variable-range hopping law [24]. Let us briefly recall how the VRH law is derived.

The VRH law mostly applies to noninteracting electrons (or Fermi liquids) in presence of phonons that can provide the inelastic scatterings needed to make transitions between states of different energies. One considers localized eigenstates at different positions in space as indicated in Fig. 1.2(a). Thanks to phonons the system can make a transition from an occupied state towards an unoccupied one. The probability of making a transition involving a difference of energy E , between two localized states at a distance L is of order

$$e^{-\beta E} e^{-L/\xi_{\text{loc}}} \quad (1.20)$$

Thus it is interesting to find transitions for states close to each other. Of course such states are not in general close in energy so there is a compromise between the energy and the distance at which one can find states. If the density of states is N_0 , then the probability in a volume L^d to find a state within an interval of energy E is

$$N_0 L^d E \quad (1.21)$$

thus a transition is possible when $N_0 L^d E \sim 1$. This leads to a conductivity that is proportional to

$$\sigma \propto e^{-\frac{\beta}{N_0 L^d} - L/\xi_{\text{loc}}} \quad (1.22)$$

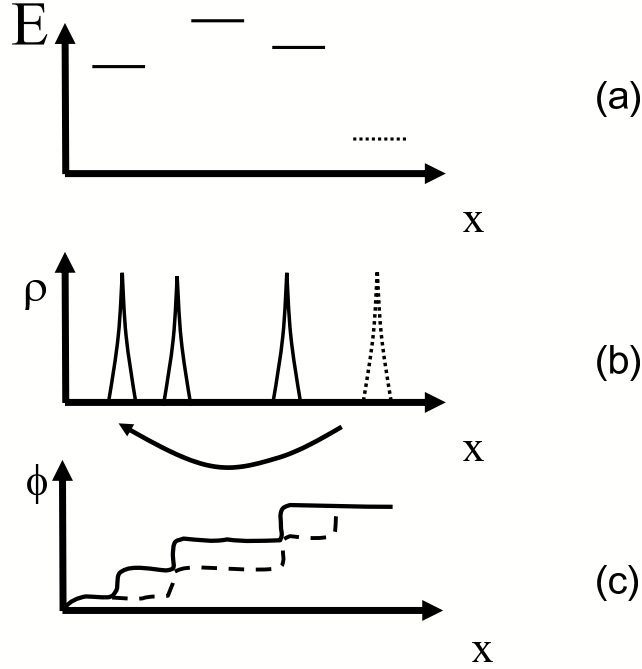


Figure 1.2. (a) If the system is very localized, the eigenstates are localized in space x over a distance of order ξ_{loc} and are spread in energy E . In order to transport current one should make a transition from an occupied state (dashed line) towards an empty state (full line). The difference in energy is provided by a coupling to a bath of phonons. This is the process at the root of the variable range hopping conductivity. (b) The density is a set of narrow peaks. Processes involved in the VRH thus transport charge from one localized state to another. (c) In the bosonized representation motion occur by shifting the phase by a multiple of π in a finite region of space (whose size is determined by optimizing the action). Using the bosonization relations (see text) this corresponds precisely to the same transport of charge than in the VRH process.

and optimizing with respect to L one finds

$$\sigma \propto e^{-(d+1) \left(\frac{\beta}{N_0 a^d \xi_{loc}^d} \right)^{\frac{1}{d+1}}} \quad (1.23)$$

In the presence of Coulomb interactions a similar formula (Efros-Shklovskii) can be derived [25, 26] but with an exponent $1/2$ instead of $1/(d+1)$. In

one dimension both give the same temperature dependence, but with different prefactors in the exponential.

In fact the tunnelling process derived from the bosonized action (1.5) is quite similar to the VRH process. Indeed using the relation between the fermion density and the phase (1.3) it is easy to see, as shown in Fig. 1.2(b) and (c) that a kink in ϕ corresponds to a fermion at that position. The instanton corresponding to a shift of ϕ over a finite region of space thus corresponds to moving a particle from a localized state to another, as in the VRH process.

The main difference is that in our approach the interactions are totally treated. This is what gives the difference of prefactors in the exponential. In our case the prefactors contains the Luttinger liquid parameters (and thus the interactions). In the case of VRH the prefactor simply depends on the density of states. Our derivation provides an alternative derivation to VRH, directly based on the bosonization representation and thus properly taking into account the effects of interactions.

6. Open issues

Of course many open issues remain. In particular, both for the intermediate temperature case and for the case of the creep, the question of the precise role of dissipation arises. In the case of the RG one assumes, in order to obtain the conductivity, that the temperature can be used as a cutoff. Doing so implicitly assumes that the system loses its coherence over a length v/T . This is reasonable if the system is in contact with an external bath, but is not the standard way to put the temperature in the Kubo formula. Indeed normally the thermal bath is applied at time $-\infty$ and then removed and all time evolution proceed simply quantum mechanically. For the case of periodic potential the two procedure are known to produce different results. If no phase breaking processes are included the system being integrable has too many conserved quantities and the conductivity remains infinite at all finite temperatures [27–30]. This rather artificial result disappears in the more realistic case where phase breaking processes are included [27, 30]. For the disordered case, in a similar way, the noninteracting system has a conductivity that would remain zero at any finite temperature since all states are exponentially localized, whereas the RG result would give a temperature dependence, as shown in Fig. 1.1. In the creep regime similar questions arise. In the VRH derivation the coupling to a phonon bath is explicitly needed to provide the energy for the transition. In our analysis such phonon bath is not needed, but on the other hand some dissipation is required for the system to reach a steady state. Whether the results we have found are valid as soon as an infinitesimal dissipation is included, or whether (1.19) would vanish with the dissipation in the limit where the dissipation goes to zero is an important open question.

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